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Estimate of Average Elastoplastic Moduli of
Composites and
Localized Deformation

A DISSERTATION

SUBMITTED TO THE DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

AND THE COMMITTEE ON GRADUATE STUDIES

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FOR THE DEGREE OF

DOCTOR OF ENGINEERING

KOJI DOBOKU

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ABSTRACT

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Koji DOBOKU

The Jaumann stress rate of the Cauchy stress is usually used to represent hypoelasticity. Since this stress rate takes into account only the effect of finite rotation; i.e. spin during motion, we here examined the effects of deformation rate terms which can be included in the definitions of the stress rates. First we have shown that the Truesdell stress rate can be defined as a rate of the 2nd Piola-Kirchhoff stress with the current state as reference; i.e. an updated Lagrangian measure.

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1 . Introduction

Analytical methods for averaging the material characteristics of composites are extremely useful in designing new materials long before carrying out either experimental trials or numerical analyses with precise models of microstructure. Among many such methods, the Mori-Tanaka approach¹⁾ is a simple one used to evaluate the average elastic and elastoplastic properties of composites²⁾. However, since the method does not take into account the mechanical interactions between many inhomogeneities, the predicted behavior, especially in the plastic states, tends to be significantly stiffer than what is observed in experiments. In order to improve its ability to predict the behavior of materials, a variety of approaches has been suggested: an explicit geometrical distribution of inhomogeneities was assumed and introduced³⁾, and secant and tangential moduli were employed to evaluate interactions approximately by Doe⁴⁾.

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2 . The averaging approach to multi-phase elastoplastic composites

(1) Mori-Tanaka averaging in incremental form

a) 3D expression

Suppose that there are $(N - 1)$ different types of ellipsoidal inhomogeneities distributed in an infinite body where the N -th phase is the matrix. Let $\dot{\sigma}$, $\dot{\varepsilon}$ and \mathbf{C} denote the incremental stress tensor, incremental strain tensor and tangential isotropic elastic tensor, respectively. Since a virtual matrix introduced in the next section is an elastic body because, for example, the Eshelby tensor can be easily evaluated, the matrix (N -th phase) is assumed to be isotropically elastic, and the corresponding constitutive relation in rate form is expressed as

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(2) Localization

a) Classical approaches

Then, based on the Mori-Tanaka approach, an approximate average constitutive relation of the matrix can be assumed by

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b) Finite deformation

The strain field of the i -th inhomogeneity must include the interaction between the particular inhomogeneity and the surrounding matrix material, and can be written as

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3 . Experimental approaches

(1) Uniaxial case

a) Infinitesimal deformation

Then the equivalent inclusion method⁵⁾ allows the following expression in the i -th inhomogeneity as

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4 . Concluding remarks

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Finally, we can define the overall average incremental stress $\dot{\bar{\sigma}}$ and the corresponding incremental strain $\dot{\bar{\epsilon}}$ of the composite by simple volume averages as

APPENDIX A . Ductility

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(2) Incremental theory

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Table 1 Material parameters

x	y
a	b
c	d

Table 2 Experimental settings

a b
c d

【図】

Fig. 1 First results

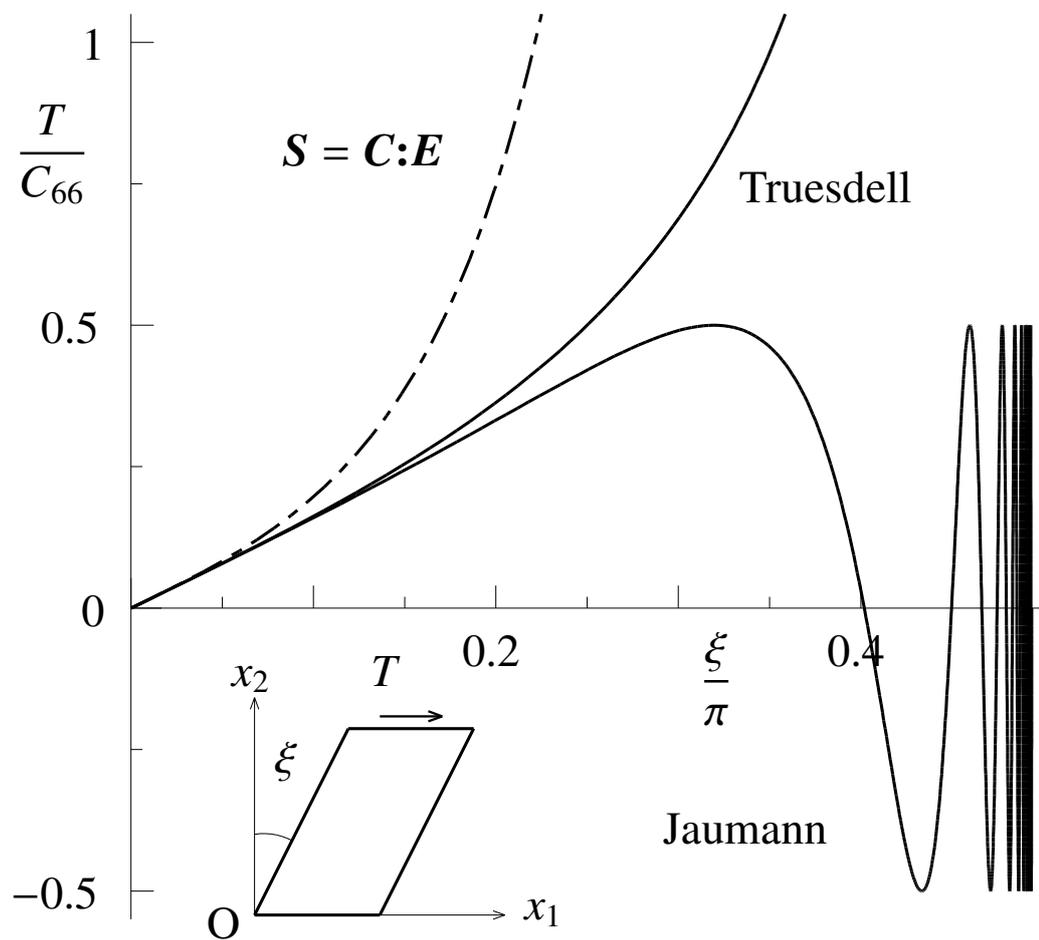


Fig. 2 Second results



Photo 1 My helicopter