Estimate of Average Elastoplastic Moduli of Composites and Localized Deformation

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ABSTRACT

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1. Introduction

Analytical methods for averaging the material characteristics of composites are extremely useful in designing new materials long before carrying out either experimental trials or numerical analyses with precise models of microstructure. Among many such methods, the Mori-Tanaka approach[1)] is a simple one used to evaluate the average elastic and elastoplastic properties of composites[2)]. However, since the method does not take into account the mechanical interactions between many inhomogeneities, the predicted behavior, especially in the plastic states, tends to be significantly stiffer than what is observed in experiments. In order to improve its ability to predict the behavior of materials, a variety of approaches has been suggested: an explicit geometrical distribution of inhomogeneities was assumed and introduced[3)], and secant and tangential moduli were employed to evaluate interactions approximately by Doe[4)].

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2 . The averaging approach to multi-phase elastoplastic composites

(1) Mori-Tanaka averaging in incremental form

a) 3D expression

Suppose that there are $(N-1)$ different types of ellipsoidal inhomogeneities distributed in an infinite body where the N-th phase is the matrix. Let $\dot{\boldsymbol{\sigma}}$, $\dot{\boldsymbol{\varepsilon}}$ and $\boldsymbol{C}$ denote the incremental stress tensor, incremental strain tensor and tangential isotropic elastic tensor, respectively. Since a virtual matrix introduced in the next section is an elastic body because, for example, the Eshelby tensor can be easily evaluated, the matrix (N-th phase) is assumed to be isotropically elastic, and the corresponding constitutive relation in rate form is expressed as

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(2) Localization

a) Classical approachs

Then, based on the Mori-Tanaka approach, an approximate average constitutive relation of the matrix can be assumed by

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b) Finite deformation

The strain field of the i-th inhomogeneity must include the interaction between the particular inhomogeneity and the surrounding matrix material, and can be written as

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3 . Experimental approaches

(1) Uniaxial case

a) Infinitesimal deformation

Then the equivalent inclusion method[5)] allows the following expression in the *i*-th inhomogeneity as

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4 . Concluding remarks

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Finally, we can define the overall average incremental stress $\dot{\bar{\boldsymbol{\sigma}}}$ and the corresponding incremental strain $\dot{\bar{\boldsymbol{\varepsilon}}}$ of the composite by simple volume averages as

APPENDIX A . Ductility

(1) In the case of ...

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(2) Incremental theory

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Table 1 Material parameters

x	y
a	b
c	d

Table 2 Experimental settings

a	b
c	d

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Fig. 1 First results

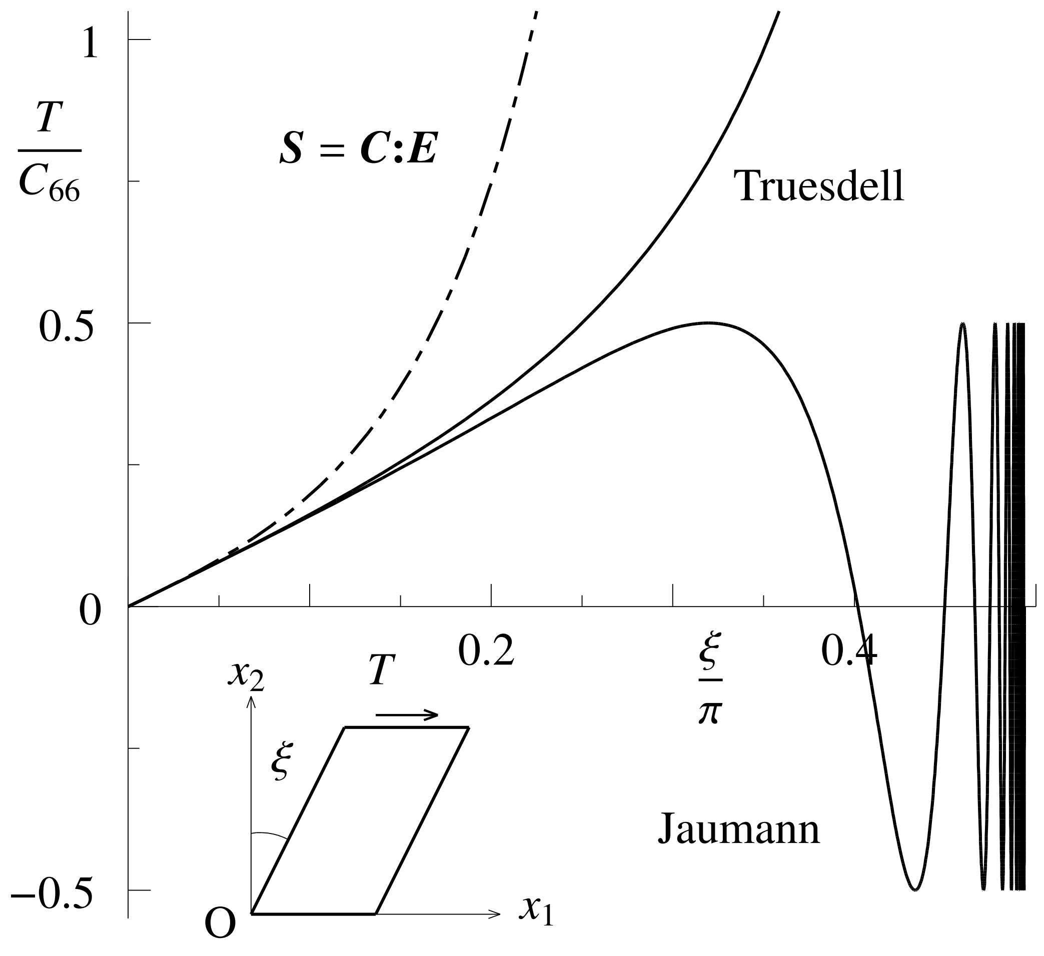

Fig. 2 Second results

Photo 1 My helicopter