卒 業 論 文

複合材料の平均的な弾塑性係数の評価と変形の局所化

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土木　浩二

2015 年 3 月

要　旨

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土木　浩二

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□△を用いた材料実験を行なった結果，ここで選んだような比較的延性を顕著に示すような材料については，かなりの精度で材料挙動を予測することが可能な構成モデルを構築することができた．さらに，増分理論に拡張することにより，有限ひずみの範囲までも無理なく拡張できた．

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謝　辞

絵来詮西教授先生にはたいへん・・・ほげほげ・・・

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1．まえがき

延性材料の破壊前には，ストライプ様のLüders帯のような周期的な絞りや，マクロなすべり線のような孤立した帯といった局所化した変形が発生することが多い。このような局所変形予測には古くはすべり線理論[1)]がよく用いられ，塑性加工の終局強度設計等に実際に利用されてきた。地盤材料等にも拡張され，活断層上の堆積層中のすべり予測[2)]等にも用いられている。これに対応した近代的な理論としては， Hillによる規準[3)]がせん断帯発生に使われる。しかし文献[4)]等では，先に周期的な絞りが発生するとされており，実験[5)]でも周期的な絞りが発生した後に孤立したせん断帯が発生している。

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2． 構成モデルと局所化条件

(1)　構成モデル

a)　3 次元的表現

例として用いる構成則には地盤や岩盤も含むことができる一般的なものを用い，各材料パラメータの影響も調べる。その代表例として Rudnicki and Rice[6)]によるモデルを用いると，

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(2)　局所化条件

a)　古典的すべり線理論 — 微小変形理論の枠組

　古典的すべり線理論は，微小変形理論の枠組の剛・完全塑性体の平面ひずみ状態で，Mises の降伏条件と Prandtl-Reuss の流れ則

　一方， Hill は平面ひずみの条件式を使わずに一軸状態を考え[7)]

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b) 有限変形理論の枠組での条件

これに対し，硬化体等のせん断帯発生を予測する理論もいくつか提案されている。その中でよく用いられるのは，均質な変形状態に発生し得る速度勾配の不連続面の発生条件[8)]だ。

3． Jaumann の応力速度を用いた場合

(1)　単純な応答

まず Jaumann の応力速度を用いた場合の，よく知られた結果を列挙しておく。材料パラメータは文献[9),10)]を参考にして Poisson 比を $\nu = 0.3$ とし，硬化域 $(H \geq 0)$ でのせん断帯発生を対象とする。

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(2)　単純な載荷状態での応答の特徴

a)　単純せん断状態

Truesdell 応力速度で構成則を定義した場合のせん断帯発生の解析をする前に， Jaumann の応力速度を用いた場合と Truesdell の応力速度を用いた場合の，その材料モデルが示す特性の違いを明らかにしておく。最初は著名な結果[11)]であるが，単純せん断をした場合の応答の違いである。**図–2** 中の小さい図にも示したように，非零の変位成分が $u_1 = X_2 \tan\xi$ のみという単純せん断を対象とする。

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4． 結論

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かなり基礎的かつ半解析的・数値的な検討に留まったが，構成則に Truesdell の応力速度を用いた場合，次のような特性が得られた。

- 単純せん断載荷と
- 予測される
- ただ，引張の局所化は

補遺 I． 延性の評価

(1)　とりあえず，まずは・・・

図 1 に示したように柔らかい介在物の導入は延性の改善を促す．ここでは，その定量的な評価を行なうために次式のような延性率 D を導入した．

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表– 1 表

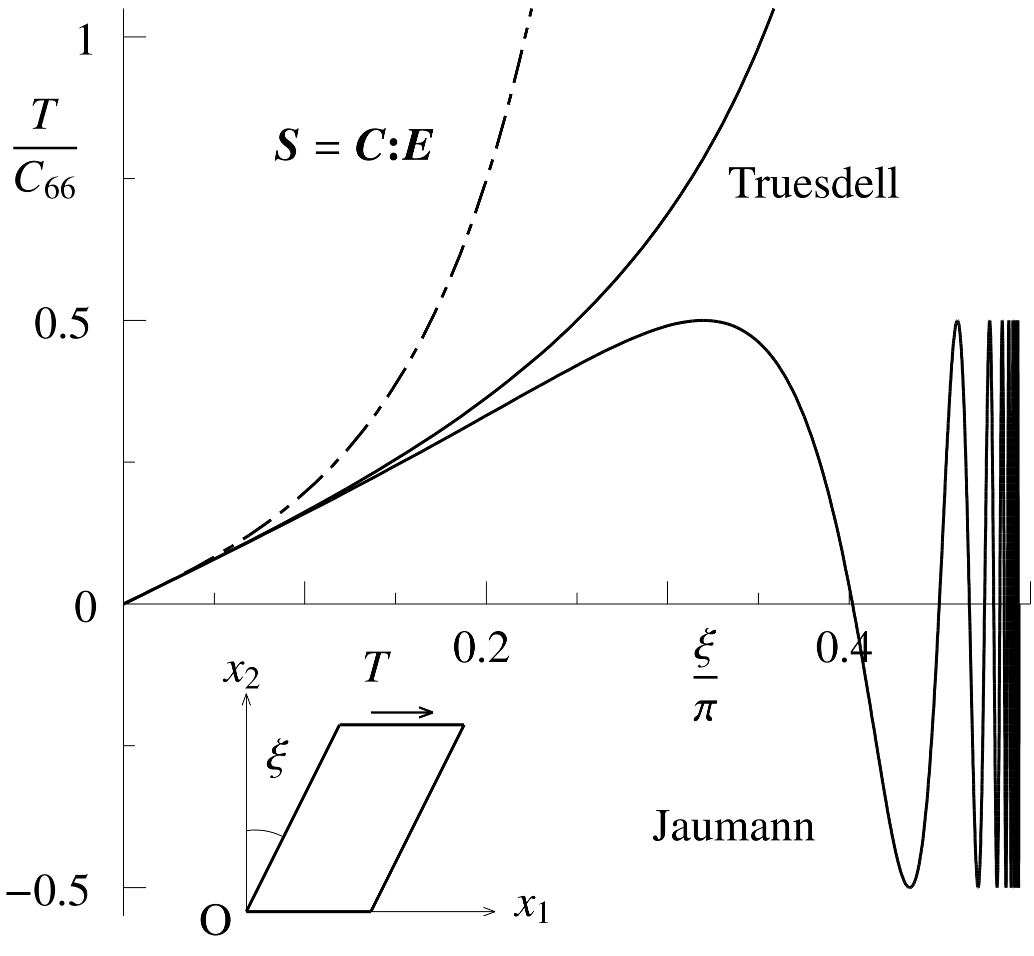
x	y
a	b
c	d

表– 2 表

a	b
c	d

【図】

図– 1 図



図– 2 弾性体の単純せん断載荷

写真–1 現場視察で利用したヘリコプター

Estimate of Average Elastoplastic Moduli of Composites and Localized Deformation

A GRADUATION THESIS

SUBMITTED TO THE DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

OF TOHOKU UNIVERSITY

FOR THE DEGREE OF

BACHELOR OF ENGINEERING

Advisor Professor Edward CIVIL

Koji DOBOKU

March 2015

ABSTRACT

ESTIMATE OF AVERAGE ELASTOPLASTIC MODULI OF COMPOSITES AND LOCALIZED DEFORMATION

Koji DOBOKU

The Jaumann stress rate of the Cauchy stress is usually used to represent hypoelasticity. Since this stress rate takes into account only the effect of finite rotation; i.e. spin during motion, we here examined the effects of deformation rate terms which can be included in the definitions of the stress rates. First we have shown that the Truesdell stress rate can be defined as a rate of the 2nd Piola-Kirchhoff stress with the current state as reference; i.e. an updated Lagrangian measure.

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ACKNOWLEDGMENT

I appreciate .....

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1 . Introduction

Analytical methods for averaging the material characteristics of composites are extremely useful in designing new materials long before carrying out either experimental trials or numerical analyses with precise models of microstructure. Among many such methods, the Mori-Tanaka approach[1)] is a simple one used to evaluate the average elastic and elastoplastic properties of composites[2)]. However, since the method does not take into account the mechanical interactions between many inhomogeneities, the predicted behavior, especially in the plastic states, tends to be significantly stiffer than what is observed in experiments. In order to improve its ability to predict the behavior of materials, a variety of approaches has been suggested: an explicit geometrical distribution of inhomogeneities was assumed and introduced[3)], and secant and tangential moduli were employed to evaluate interactions approximately by Doe[4)].

The Jaumann stress rate of the Cauchy stress is usually used to represent hypoelasticity. Since this stress rate takes into account only the effect of finite rotation; i.e. spin during motion, we here examined the effects of deformation rate terms which can be included in the definitions of the stress rates. First we have shown that the Truesdell stress rate can be defined as a rate of the 2nd Piola-Kirchhoff stress with the current state as reference; i.e. an updated Lagrangian measure.

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2 . The averaging approach to multi-phase elastoplastic composites

(1) Mori-Tanaka averaging in incremental form

a) 3D expression

Suppose that there are $(N - 1)$ different types of ellipsoidal inhomogeneities distributed in an infinite body where the N-th phase is the matrix. Let $\dot{\boldsymbol{\sigma}}$, $\dot{\boldsymbol{\varepsilon}}$ and $\boldsymbol{C}$ denote the incremental stress tensor, incremental strain tensor and tangential isotropic elastic tensor, respectively. Since a virtual matrix introduced in the next section is an elastic body because, for example, the Eshelby tensor can be easily evaluated, the matrix (N-th phase) is assumed to be isotropically elastic, and the corresponding constitutive relation in rate form is expressed as

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(2) Localization

a) Classical approachs

Then, based on the Mori-Tanaka approach, an approximate average constitutive relation of the matrix can be assumed by

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b) Finite deformation

The strain field of the i-th inhomogeneity must include the interaction between the particular inhomogeneity and the surrounding matrix material, and can be written as

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3 . Experimental approaches

(1) Uniaxial case

a) Infinitesimal deformation

Then the equivalent inclusion method[5] allows the following expression in the *i*-th inhomogeneity as

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4 . Concluding remarks

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Finally, we can define the overall average incremental stress $\dot{\bar{\boldsymbol{\sigma}}}$ and the corresponding incremental strain $\dot{\bar{\boldsymbol{\varepsilon}}}$ of the composite by simple volume averages as

APPENDIX A . Ductility

(1) In the case of ...

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(2) Incremental theory

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Table 1 Material parameters

x	y
a	b
c	d

Table 2 Experimental settings

a	b
c	d

【図】

Fig. 1 First results

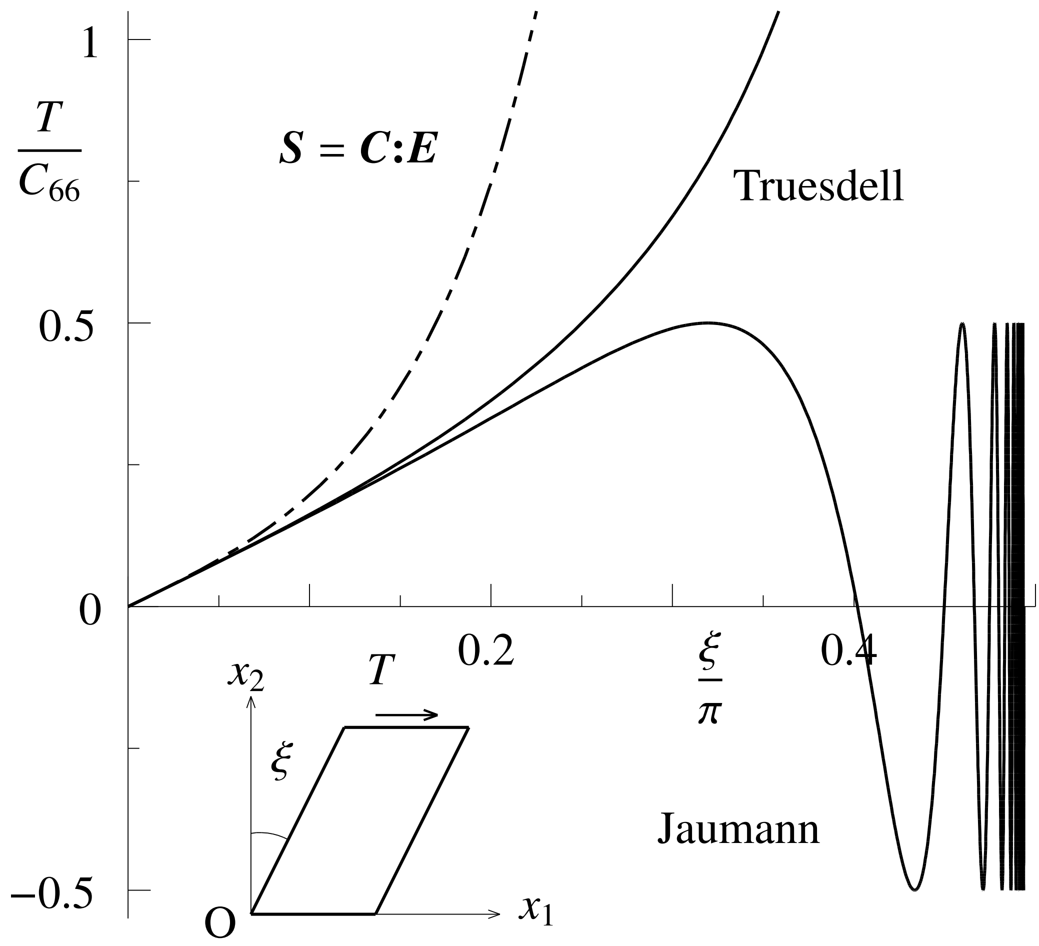

Fig. 2 Second results

Photo 1 My helicopter

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研究指導教員	僕蛾　教絵太　准教授
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【学歴】

平成 10 年 4 月 1 日	東北大学工学部	入学
平成 14 年 3 月 26 日	同	卒業
平成 14 年 4 月 1 日	東北大学大学院工学研究科博士課程前期 2 年の課程	入学
平成 16 年 3 月 25 日	同	修了
平成 23 年 4 月 1 日	東北大学大学院工学研究科博士課程後期 3 年の課程	編入学
平成 26 年 3 月 26 日	同	修了

【職歴】

平成 16 年 4 月 1 日	何出模建設株式会社　入社

備考 (1) 外国人留学生は，国籍を記入すること。

(2) 履歴事項は，大学入学から年次にしたがって記入すること。

修 士 論 文

複合材料の平均的な弾塑性係数の評価と変形の局所化

Estimate of Average Elastoplastic Moduli of Composites and Localized Deformation

東北大学大学院工学研究科土木工学専攻

土木　浩二

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謝　辞

絵来詮西教授先生にはたいへん・・・ほげほげ・・・

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1． まえがき

延性材料の破壊前には，ストライプ様の Lüders 帯のような周期的な絞りや，マクロなすべり線のような孤立した帯といった局所化した変形が発生することが多い。このような局所変形予測には古くはすべり線理論[1)]がよく用いられ，塑性加工の終局強度設計等に実際に利用されてきた。地盤材料等にも拡張され，活断層上の堆積層中のすべり予測[2)]等にも用いられている。これに対応した近代的な理論としては， Hill による規準[3)]がせん断帯発生に使われる。しかし文献[4)]等では，先に周期的な絞りが発生するとされており，実験[5)]でも周期的な絞りが発生した後に孤立したせん断帯が発生している。

非均質材料の平均的な弾性挙動の予測に用いられている簡単な方法を拡張し，微小ひずみの範囲での複合材料の弾塑性挙動の把握を行なった．□△を用いた材料実験を行なった結果，ここで選んだような比較的延性を顕著に示すような材料については，かなりの精度で材料挙動を予測することが可能な構成モデルを構築することができた．さらに，増分理論に拡張することにより，有限ひずみの範囲までも無理なく拡張できた．

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2．構成モデルと局所化条件

(1)　構成モデル

a)　3 次元的表現

例として用いる構成則には地盤や岩盤も含むことができる一般的なものを用い，各材料パラメータの影響も調べる。その代表例として Rudnicki and Rice[6)] によるモデルを用いると，

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(2)　局所化条件

a)　古典的すべり線理論 — 微小変形理論の枠組

古典的すべり線理論は，微小変形理論の枠組の剛・完全塑性体の平面ひずみ状態で，Mises の降伏条件と Prandtl-Reuss の流れ則

一方， Hill は平面ひずみの条件式を使わずに一軸状態を考え[7)]

非均質材料の平均的な弾性挙動の予測に用いられている簡単な方法を拡張し，微小ひずみの範囲での複合材料の弾塑性挙動の把握を行なった．□△を用いた材料実験を行なった結果，ここで選んだような比較的延性を顕著に示すような材料については，かなりの精度で材料挙動を予測することが可能な構成モデルを構築することができた．さらに，増分理論に拡張することにより，有限ひずみの範囲までも無理なく拡張できた．

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た結果，ここで選んだような比較的延性を顕著に示すような材料については，かなりの精度で材料挙動を予測することが可能な構成モデルを構築することができた．さらに，増分理論に拡張することにより，有限ひずみの範囲までも無理なく拡張できた．

b)　有限変形理論の枠組での条件

これに対し，硬化体等のせん断帯発生を予測する理論もいくつか提案されている。その中でよく用いられるのは，均質な変形状態に発生し得る速度勾配の不連続面の発生条件[8)]だ。

3． Jaumann の応力速度を用いた場合

(1)　単純な応答

まず Jaumann の応力速度を用いた場合の，よく知られた結果を列挙しておく。材料パラメータは文献[9),10)]を参考にして Poisson 比を $\nu = 0.3$ とし，硬化域 $(H \geq 0)$ でのせん断帯発生を対象とする。

非均質材料の平均的な弾性挙動の予測に用いられている簡単な方法を拡張し，微小ひずみの範囲での複合材料の弾塑性挙動の把握を行なった．□△を用いた材料実験を行なった結果，ここで選んだような比較的延性を顕著に示すような材料については，かなりの精度で材料挙動を予測することが可能な構成モデルを構築することができた．さらに，増分理論に拡張することにより，有限ひずみの範囲までも無理なく拡張できた．

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精度で材料挙動を予測することが可能な構成モデルを構築することができた．さらに，増分理論に拡張することにより，有限ひずみの範囲までも無理なく拡張できた．

(2)　単純な載荷状態での応答の特徴

a)　単純せん断状態

Truesdell 応力速度で構成則を定義した場合のせん断帯発生の解析をする前に， Jaumann の応力速度を用いた場合と Truesdell の応力速度を用いた場合の，その材料モデルが示す特性の違いを明らかにしておく。最初は著名な結果[11)]であるが，単純せん断をした場合の応答の違いである。**図–2** 中の小さい図にも示したように，非零の変位成分が $u_1 = X_2 \tan\xi$ のみという単純せん断を対象とする。

非均質材料の平均的な弾性挙動の予測に用いられている簡単な方法を拡張し，微小ひずみの範囲での複合材料の弾塑性挙動の把握を行なった．□△を用いた材料実験を行なった結果，ここで選んだような比較的延性を顕著に示すような材料については，かなりの精度で材料挙動を予測することが可能な構成モデルを構築することができた．さらに，増分理論に拡張することにより，有限ひずみの範囲までも無理なく拡張できた．

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4． 結論

非均質材料の平均的な弾性挙動の予測に用いられている簡単な方法を拡張し，微小ひずみの範囲での複合材料の弾塑性挙動の把握を行なった．□△を用いた材料実験を行なった結果，ここで選んだような比較的延性を顕著に示すような材料については，かなりの精度で材料挙動を予測することが可能な構成モデルを構築することができた．さらに，増分理論に拡張することにより，有限ひずみの範囲までも無理なく拡張できた．

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かなり基礎的かつ半解析的・数値的な検討に留まったが，構成則に Truesdell の応力速度を用いた場合，次のような特性が得られた。

- 単純せん断載荷と
- 予測される
- ただ，引張の局所化は

補遺 I．延性の評価

(1)　とりあえず，まずは・・・

図 1 に示したように柔らかい介在物の導入は延性の改善を促す．ここでは，その定量的な評価を行なうために次式のような延性率 D を導入した．

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表– 1 表

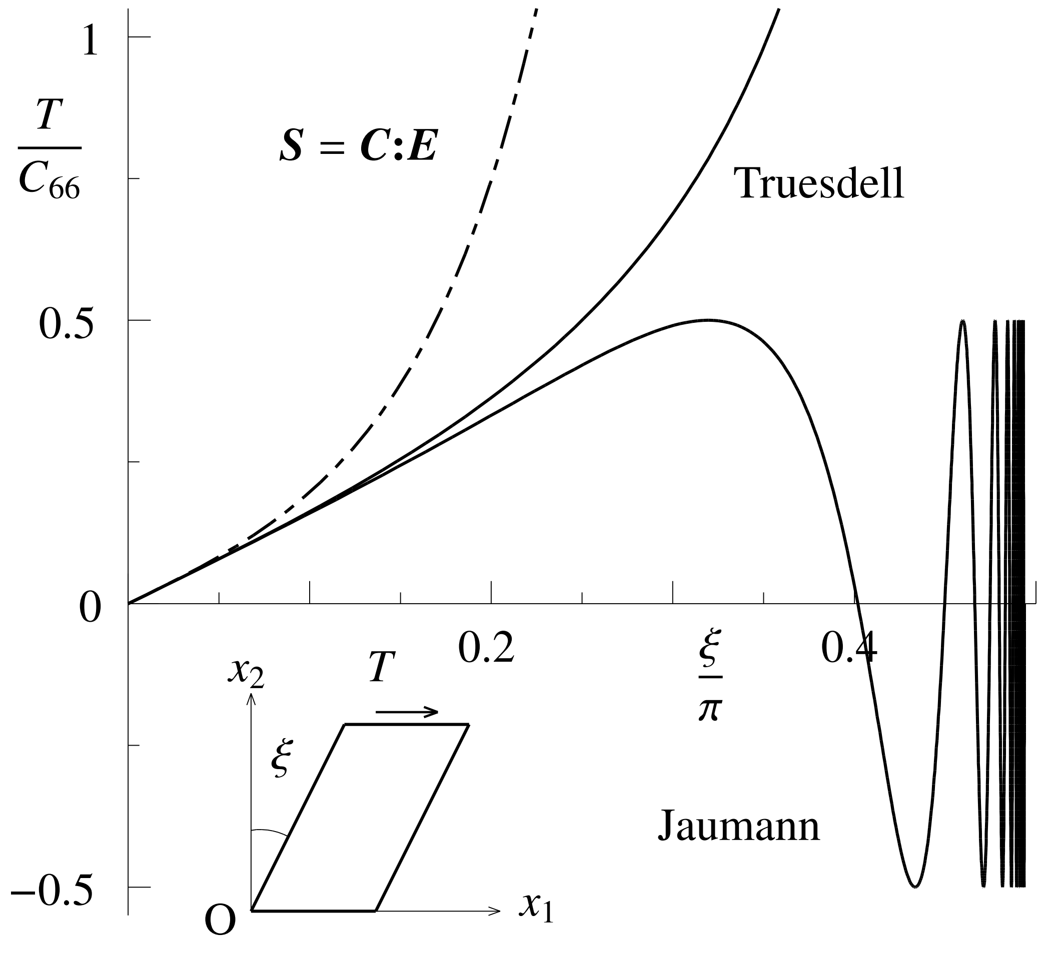
x	y
a	b
c	d

表– 2 表

a	b
c	d

【図】

図–1 図



図–2 弾性体の単純せん断載荷

写真–1 現場視察で利用したヘリコプター

Advising Professor at Tohoku Univ.	Professor Edward CIVIL
Research Advisor at Tohoku Univ.	Assoc.Prof. J. DOE
Dissertation Committee Members Name marked with "○" is the Chief Examiner	○ Professor Edward CIVIL 1 Prof. W. DOE 2 Prof. L. DOE 3 Prof. T. DOE 4 Assoc.Prof. J. DOE

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From To	
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From To	
From To	

Note: Educational background has to be filled in, starting from the date of university enrollment.

Estimate of Average Elastoplastic Moduli of Composites and Localized Deformation

A MASTER THESIS

SUBMITTED TO THE DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

AND THE COMMITTEE ON GRADUATE STUDIES

OF TOHOKU UNIVERSITY

FOR THE DEGREE OF

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Koji DOBOKU

March 2015

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ABSTRACT

ESTIMATE OF AVERAGE ELASTOPLASTIC MODULI OF COMPOSITES AND LOCALIZED DEFORMATION

Koji DOBOKU

The Jaumann stress rate of the Cauchy stress is usually used to represent hypoelasticity. Since this stress rate takes into account only the effect of finite rotation; i.e. spin during motion, we here examined the effects of deformation rate terms which can be included in the definitions of the stress rates. First we have shown that the Truesdell stress rate can be defined as a rate of the 2nd Piola-Kirchhoff stress with the current state as reference; i.e. an updated Lagrangian measure.

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ACKNOWLEDGMENT

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1 . Introduction

Analytical methods for averaging the material characteristics of composites are extremely useful in designing new materials long before carrying out either experimental trials or numerical analyses with precise models of microstructure. Among many such methods, the Mori-Tanaka approach[1)] is a simple one used to evaluate the average elastic and elastoplastic properties of composites[2)]. However, since the method does not take into account the mechanical interactions between many inhomogeneities, the predicted behavior, especially in the plastic states, tends to be significantly stiffer than what is observed in experiments. In order to improve its ability to predict the behavior of materials, a variety of approaches has been suggested: an explicit geometrical distribution of inhomogeneities was assumed and introduced[3)], and secant and tangential moduli were employed to evaluate interactions approximately by Doe[4)].

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2 . The averaging approach to multi-phase elastoplastic composites

(1) Mori-Tanaka averaging in incremental form

a) 3D expression

Suppose that there are $(N-1)$ different types of ellipsoidal inhomogeneities distributed in an infinite body where the N-th phase is the matrix. Let $\dot{\boldsymbol{\sigma}}$, $\dot{\boldsymbol{\varepsilon}}$ and $\boldsymbol{C}$ denote the incremental stress tensor, incremental strain tensor and tangential isotropic elastic tensor, respectively. Since a virtual matrix introduced in the next section is an elastic body because, for example, the Eshelby tensor can be easily evaluated, the matrix (N-th phase) is assumed to be isotropically elastic, and the corresponding constitutive relation in rate form is expressed as

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(2) Localization

a) Classical approachs

Then, based on the Mori-Tanaka approach, an approximate average constitutive relation of the matrix can be assumed by

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b) Finite deformation

The strain field of the i-th inhomogeneity must include the interaction between the particular inhomogeneity and the surrounding matrix material, and can be written as

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3 . Experimental approaches

(1) Uniaxial case

a) Infinitesimal deformation

Then the equivalent inclusion method[5)] allows the following expression in the *i*-th inhomogeneity as

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4 . Concluding remarks

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Finally, we can define the overall average incremental stress $\dot{\bar{\boldsymbol{\sigma}}}$ and the corresponding incremental strain $\dot{\bar{\boldsymbol{\varepsilon}}}$ of the composite by simple volume averages as

APPENDIX A . Ductility

(1) In the case of ...

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(2) Incremental theory

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Table 1 Material parameters

x	y
a	b
c	d

Table 2 Experimental settings

a	b
c	d

【図】

Fig. 1 First results

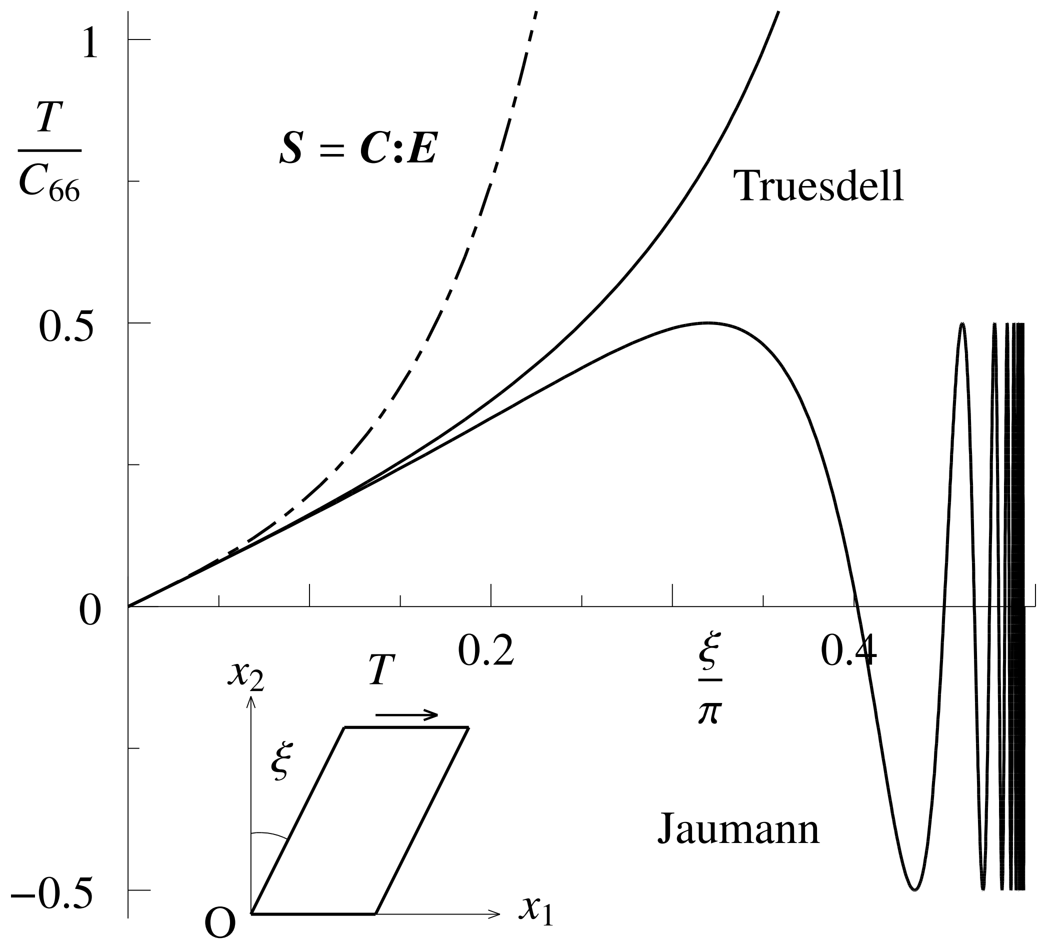

Fig. 2 Second results

Photo 1 My helicopter

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平成16年3月25日	同　修了
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備考 (1) 外国人留学生は，国籍を記入すること。

(2) 履歴事項は，大学入学から年次にしたがって記入すること。

博 士 論 文

複合材料の平均的な弾塑性係数の評価と変形の局所化

Estimate of Average Elastoplastic Moduli of Composites and Localized Deformation

東北大学大学院工学研究科土木工学専攻

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2015 年 3 月

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謝　辞

絵来詮西教授先生にはたいへん・・・ほげほげ・・・

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1． まえがき

延性材料の破壊前には，ストライプ様の Lüders 帯のような周期的な絞りや，マクロなすべり線のような孤立した帯といった局所化した変形が発生することが多い。このような局所変形予測には古くはすべり線理論[1)]がよく用いられ，塑性加工の終局強度設計等に実際に利用されてきた。地盤材料等にも拡張され，活断層上の堆積層中のすべり予測[2)]等にも用いられている。これに対応した近代的な理論としては， Hill による規準[3)]がせん断帯発生に使われる。しかし文献[4)]等では，先に周期的な絞りが発生するとされており，実験[5)]でも周期的な絞りが発生した後に孤立したせん断帯が発生している。

非均質材料の平均的な弾性挙動の予測に用いられている簡単な方法を拡張し，微小ひずみの範囲での複合材料の弾塑性挙動の把握を行なった．□△を用いた材料実験を行なった結果，ここで選んだような比較的延性を顕著に示すような材料については，かなりの精度で材料挙動を予測することが可能な構成モデルを構築することができた．さらに，増分理論に拡張することにより，有限ひずみの範囲までも無理なく拡張できた．

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2． 構成モデルと局所化条件

(1) 構成モデル

a) 3 次元的表現

例として用いる構成則には地盤や岩盤も含むことができる一般的なものを用い，各材料パラメータの影響も調べる。その代表例として Rudnicki and Rice[6)]によるモデルを用いると，

非均質材料の平均的な弾性挙動の予測に用いられている簡単な方法を拡張し，微小ひずみの範囲での複合材料の弾塑性挙動の把握を行なった．□△を用いた材料実験を行なった結果，ここで選んだような比較的延性を顕著に示すような材料については，かなりの精度で材料挙動を予測することが可能な構成モデルを構築することができた．さらに，増分理論に拡張することにより，有限ひずみの範囲までも無理なく拡張できた．

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(2)　局所化条件

a)　古典的すべり線理論 — 微小変形理論の枠組

古典的すべり線理論は，微小変形理論の枠組の剛・完全塑性体の平面ひずみ状態で，Mises の降伏条件と Prandtl-Reuss の流れ則

一方， Hill は平面ひずみの条件式を使わずに一軸状態を考え[7)]

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b)　有限変形理論の枠組での条件

これに対し，硬化体等のせん断帯発生を予測する理論もいくつか提案されている。その中でよく用いられるのは，均質な変形状態に発生し得る速度勾配の不連続面の発生条件[8)]だ。

3． Jaumann の応力速度を用いた場合

(1)　単純な応答

まず Jaumann の応力速度を用いた場合の，よく知られた結果を列挙しておく。材料パラメータは文献[9),10)]を参考にして Poisson 比を $\nu = 0.3$ とし，硬化域 $(H \geq 0)$ でのせん断帯発生を対象とする。

非均質材料の平均的な弾性挙動の予測に用いられている簡単な方法を拡張し，微小ひずみの範囲での複合材料の弾塑性挙動の把握を行なった．□△を用いた材料実験を行なった結果，ここで選んだような比較的延性を顕著に示すような材料については，かなりの精度で材料挙動を予測することが可能な構成モデルを構築することができた．さらに，増分理論に拡張することにより，有限ひずみの範囲までも無理なく拡張できた．

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精度で材料挙動を予測することが可能な構成モデルを構築することができた．さらに，増分理論に拡張することにより，有限ひずみの範囲までも無理なく拡張できた．

(2)　単純な載荷状態での応答の特徴

a)　単純せん断状態

Truesdell 応力速度で構成則を定義した場合のせん断帯発生の解析をする前に， Jaumann の応力速度を用いた場合と Truesdell の応力速度を用いた場合の，その材料モデルが示す特性の違いを明らかにしておく。最初は著名な結果[11)]であるが，単純せん断をした場合の応答の違いである。**図–2** 中の小さい図にも示したように，非零の変位成分が $u_1 = X_2 \tan\xi$ のみという単純せん断を対象とする。

非均質材料の平均的な弾性挙動の予測に用いられている簡単な方法を拡張し，微小ひずみの範囲での複合材料の弾塑性挙動の把握を行なった．□△を用いた材料実験を行なった結果，ここで選んだような比較的延性を顕著に示すような材料については，かなりの精度で材料挙動を予測することが可能な構成モデルを構築することができた．さらに，増分理論に拡張することにより，有限ひずみの範囲までも無理なく拡張できた．

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4． 結論

非均質材料の平均的な弾性挙動の予測に用いられている簡単な方法を拡張し，微小ひずみの範囲での複合材料の弾塑性挙動の把握を行なった．□△を用いた材料実験を行なった結果，ここで選んだような比較的延性を顕著に示すような材料については，かなりの精度で材料挙動を予測することが可能な構成モデルを構築することができた．さらに，増分理論に拡張することにより，有限ひずみの範囲までも無理なく拡張できた．

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かなり基礎的かつ半解析的・数値的な検討に留まったが，構成則に Truesdell の応力速度を用いた場合，次のような特性が得られた。

- 単純せん断載荷と
- 予測される
- ただ，引張の局所化は

補遺 I． 延性の評価

(1)　とりあえず，まずは・・・

図 1 に示したように柔らかい介在物の導入は延性の改善を促す．ここでは，その定量的な評価を行なうために次式のような延性率 D を導入した．

非均質材料の平均的な弾性挙動の予測に用いられている簡単な方法を拡張し，微小ひずみの範囲での複合材料の弾塑性挙動の把握を行なった．□△を用いた材料実験を行なった結果，ここで選んだような比較的延性を顕著に示すような材料については，かなりの精度で材料挙動を予測することが可能な構成モデルを構築することができた．さらに，増分理論に拡張することにより，有限ひずみの範囲までも無理なく拡張できた．

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表– 1 表

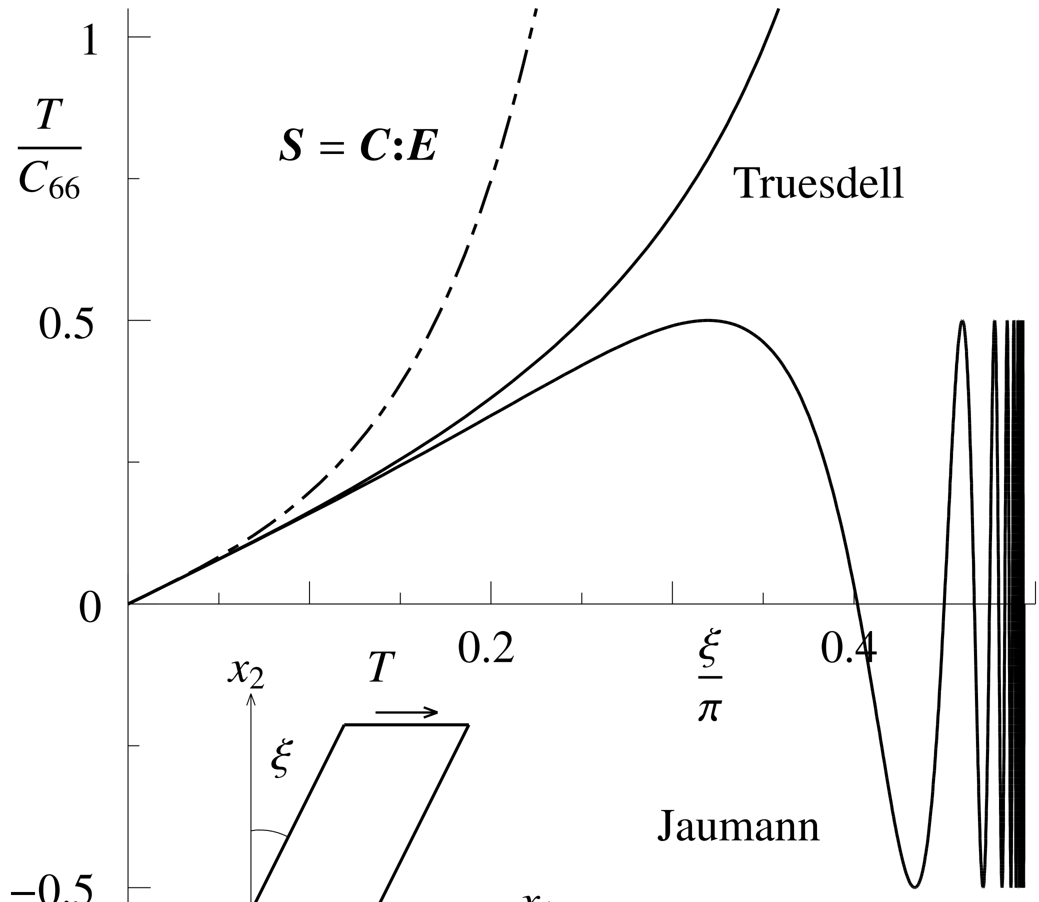
x	y
a	b
c	d

表– 2 表

a	b
c	d

【図】

図– 1 図



図– 2 弾性体の単純せん断載荷

写真–1 現場視察で利用したヘリコプター

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Research Advisor at Tohoku Univ.	Assoc.Prof. J. DOE
Dissertation Committee Members Name marked with "○" is the Chief Examiner	○ Professor Edward CIVIL 1 Prof. W. DOE 2 Prof. L. DOE 3 Prof. T. DOE 4 Assoc.Prof. J. DOE

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From To	
From To	

Note: Educational background has to be filled in, starting from the date of university enrollment.

Estimate of Average Elastoplastic Moduli of Composites and Localized Deformation

A DISSERTATION

SUBMITTED TO THE DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

AND THE COMMITTEE ON GRADUATE STUDIES

OF TOHOKU UNIVERSITY

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

DOCTOR OF ENGINEERING

Koji DOBOKU

March 2015

Professor Edward CIVIL
(Principal Advisor)

Prof. W. DOE

Prof. L. DOE

Prof. T. DOE

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ABSTRACT

ESTIMATE OF AVERAGE ELASTOPLASTIC MODULI OF COMPOSITES AND LOCALIZED DEFORMATION

Koji DOBOKU

The Jaumann stress rate of the Cauchy stress is usually used to represent hypoelasticity. Since this stress rate takes into account only the effect of finite rotation; i.e. spin during motion, we here examined the effects of deformation rate terms which can be included in the definitions of the stress rates. First we have shown that the Truesdell stress rate can be defined as a rate of the 2nd Piola-Kirchhoff stress with the current state as reference; i.e. an updated Lagrangian measure.

In order to compare the characteristics of the stress rates, localization of deformation was predicted by using the Truesdell stress rate and the convected stress rate, and the results were compared with those by the Jaumann stress rate of the Cauchy stress. However, in plane strain state, the predicted stresses of incipience of the localization by the Truesdell stress rate become close to the experimental critical stresses, Also, the orientations of the localized deformation obtained by the Truesdell stress rate showed consistency with those by the infinitesimal deformation theory, when the stress levels of the localization were in practical order.

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ACKNOWLEDGMENT

I appreciate .....

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1. Introduction

Analytical methods for averaging the material characteristics of composites are extremely useful in designing new materials long before carrying out either experimental trials or numerical analyses with precise models of microstructure. Among many such methods, the Mori-Tanaka approach[1)] is a simple one used to evaluate the average elastic and elastoplastic properties of composites[2)]. However, since the method does not take into account the mechanical interactions between many inhomogeneities, the predicted behavior, especially in the plastic states, tends to be significantly stiffer than what is observed in experiments. In order to improve its ability to predict the behavior of materials, a variety of approaches has been suggested: an explicit geometrical distribution of inhomogeneities was assumed and introduced[3)], and secant and tangential moduli were employed to evaluate interactions approximately by Doe[4)].

The Jaumann stress rate of the Cauchy stress is usually used to represent hypoelasticity. Since this stress rate takes into account only the effect of finite rotation; i.e. spin during motion, we here examined the effects of deformation rate terms which can be included in the definitions of the stress rates. First we have shown that the Truesdell stress rate can be defined as a rate of the 2nd Piola-Kirchhoff stress with the current state as reference; i.e. an updated Lagrangian measure.

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2 . The averaging approach to multi-phase elastoplastic composites

(1) Mori-Tanaka averaging in incremental form

a) 3D expression

Suppose that there are $(N-1)$ different types of ellipsoidal inhomogeneities distributed in an infinite body where the N-th phase is the matrix. Let $\dot{\boldsymbol{\sigma}}$, $\dot{\boldsymbol{\varepsilon}}$ and $\boldsymbol{C}$ denote the incremental stress tensor, incremental strain tensor and tangential isotropic elastic tensor, respectively. Since a virtual matrix introduced in the next section is an elastic body because, for example, the Eshelby tensor can be easily evaluated, the matrix (N-th phase) is assumed to be isotropically elastic, and the corresponding constitutive relation in rate form is expressed as

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(2) Localization

a) Classical approachs

Then, based on the Mori-Tanaka approach, an approximate average constitutive relation of the matrix can be assumed by

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b) Finite deformation

The strain field of the *i*-th inhomogeneity must include the interaction between the particular inhomogeneity and the surrounding matrix material, and can be written as

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3 . Experimental approaches

(1) Uniaxial case

a) Infinitesimal deformation

Then the equivalent inclusion method[5)] allows the following expression in the *i*-th inhomogeneity as

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4 . Concluding remarks

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In order to compare the characteristics of the stress rates, localization of deformation

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Finally, we can define the overall average incremental stress $\dot{\bar{\boldsymbol{\sigma}}}$ and the corresponding incremental strain $\dot{\bar{\boldsymbol{\varepsilon}}}$ of the composite by simple volume averages as

APPENDIX A . Ductility

(1) In the case of ...

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(2) Incremental theory

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Table 1 Material parameters

x	y
a	b
c	d

Table 2 Experimental settings

a	b
c	d

【図】

Fig. 1 First results

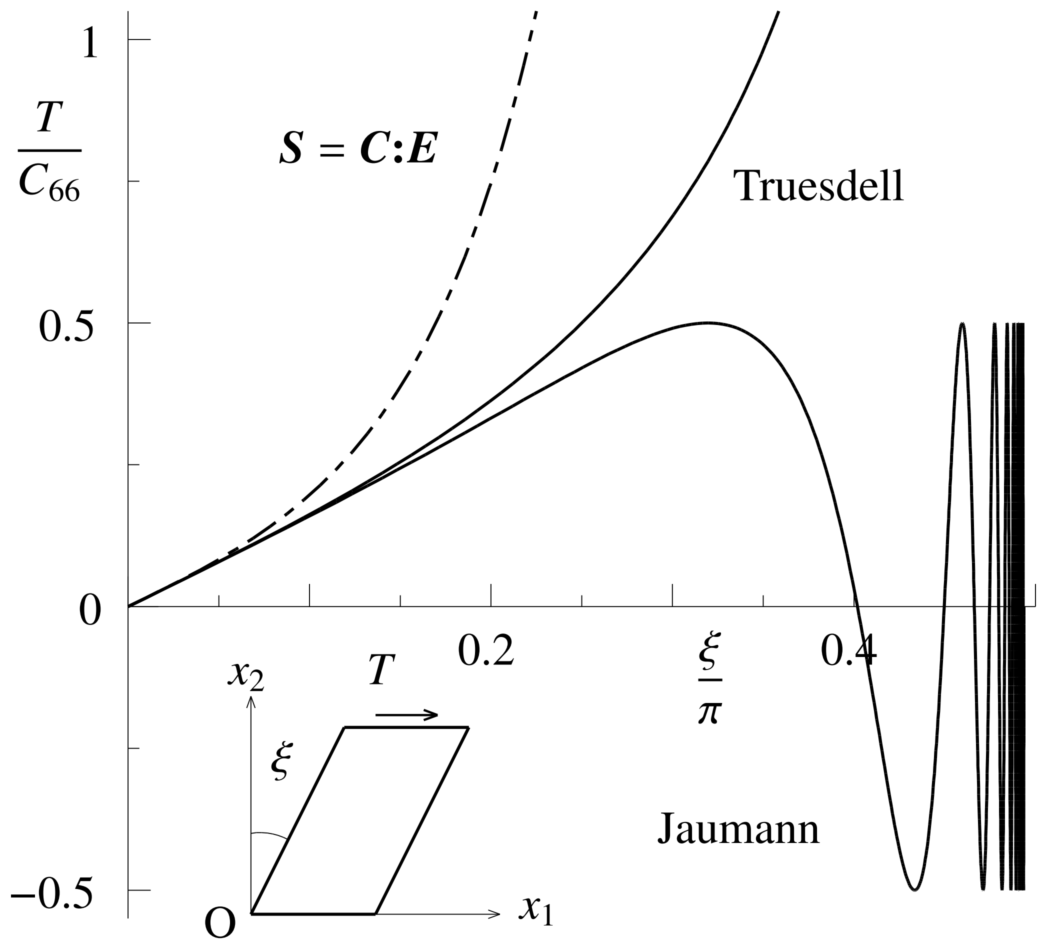

Fig. 2 Second results

Photo 1 My helicopter